Rossmoyne Senior High School

MATHEMATICS APPLICATIONS UNIT 3
Section One:
Calculator-free

SOLUTIONS

Student number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	10	10	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

       SN085-116-3

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(10 marks)**

(a) Determine the next two terms for the following sequences.

(i) $A_{n+1} = 2A_n + 3$ where $A_1 = -5$ (2 marks)

Solution
$A_2 = -7$, $A_3 = -11$
Specific behaviours
✓ determines A_2
✓ determines A_3

(ii) $P_n = \dfrac{P_{n-1}}{2} + 2^n$ where $P_1 = 2$ (2 marks)

Solution
$P_2 = 5$, $P_3 = 10\cdot5$
Specific behaviours
✓ determines P_2
✓ determines P_3

(b) Given $T_{n+1} = 5 - T_n$ where $T_3 = -13$, find the first two terms. (2 marks)

Solution	
$T_3 = 5 - T_2$	$T_2 = 5 - T_1$
$-13 = 5 - T_2$	$18 = 5 - T_1$
$-18 = -T_2$	$13 = -T_1$
$18 = T_2$	$-13 = T_1$
Specific behaviours	
✓ determines T_2	
✓ determines T_1	

(c) Determine the recursive definition for the following. (4 marks)

(i) $\dfrac{1}{2}, \dfrac{5}{4}, \dfrac{4}{2}, \dfrac{11}{4}, \ldots$

Solution
$T_{n+1} = T_n + \dfrac{3}{4}$ $\qquad T_1 = \dfrac{1}{2}$
Specific behaviours
✓ writes the rule
✓ writes first term

(ii) 81, 27, 9, ……

Solution
$T_{n+1} = \dfrac{T_n}{3}$ $\qquad T_1 = 81$
Specific behaviours
✓ writes a rule
✓ writes first term

Question 2   **(4 marks)**

The first five terms of an arithmetic sequence are shown on the graph below.
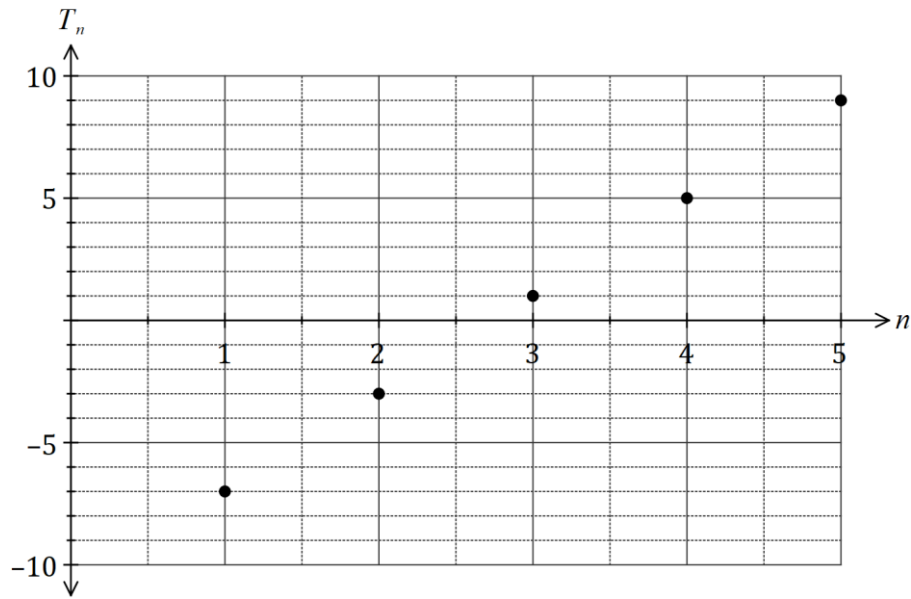
(a)   Deduce a rule for the nth term of this sequence.   (2 marks)

Solution
$T_n = -7 + (n-1)(4)$ $= 4n - 11$
Specific behaviours
✓ identifies common difference   ✓ writes a rule (accept any correct form)

(b)   Given that the kth term of this sequence is 1 197, determine the value of k.   (2 marks)
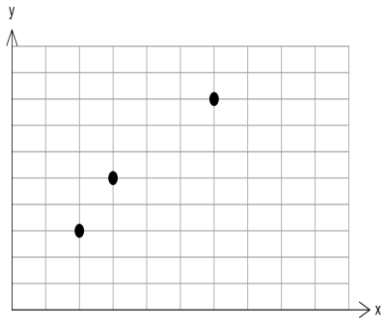
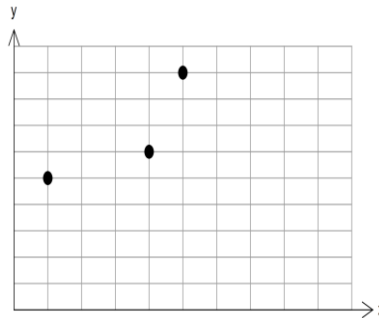| Solution |
|---|
| $-7 + (k-1)(4) = 1197$
 $(k-1)(4) = 1204$

 $k - 1 = \dfrac{1204}{4} = 301$
 $k = 302$ |
| **Specific behaviours** |
| ✓ uses equation from (a)
 ✓ solves for k |

Question 3 (6 marks)

(a) Match the following correlation coefficient values, –0.8, 0.95, 0.76, to the
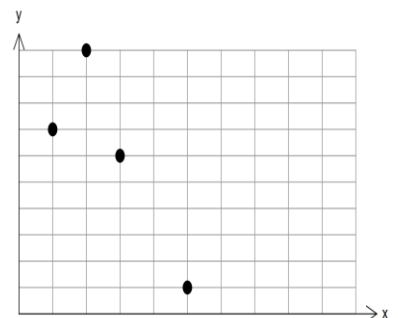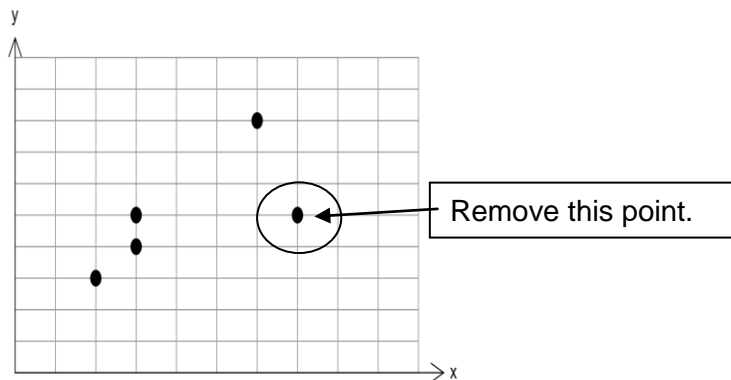graphs below. Write the value next to the letter. (3 marks)

A **B** **C**

Solution		
A = 0.95	B = 0.76	C = –0.8
Specific behaviours		
✓ correct value for A	✓ correct value for B	✓ correct value for C

(b) The graph below represents a correlation coefficient of 0.68. Circle the point
that needs to be removed to make the correlation coefficient value higher. (1 mark)

Remove this point.

Solution
See diagram
Specific behaviours
✓ clearly circles or indicates the point

(c) Students in a high school were asked what their preferred mode of transport was
for travelling to school. Column percentages are shown. Comment on the association
that exists between the variables. (2 marks)

	Girls	Boys
Car	45	27
Bike	3	25
Bus	28	28
Walk	24	20

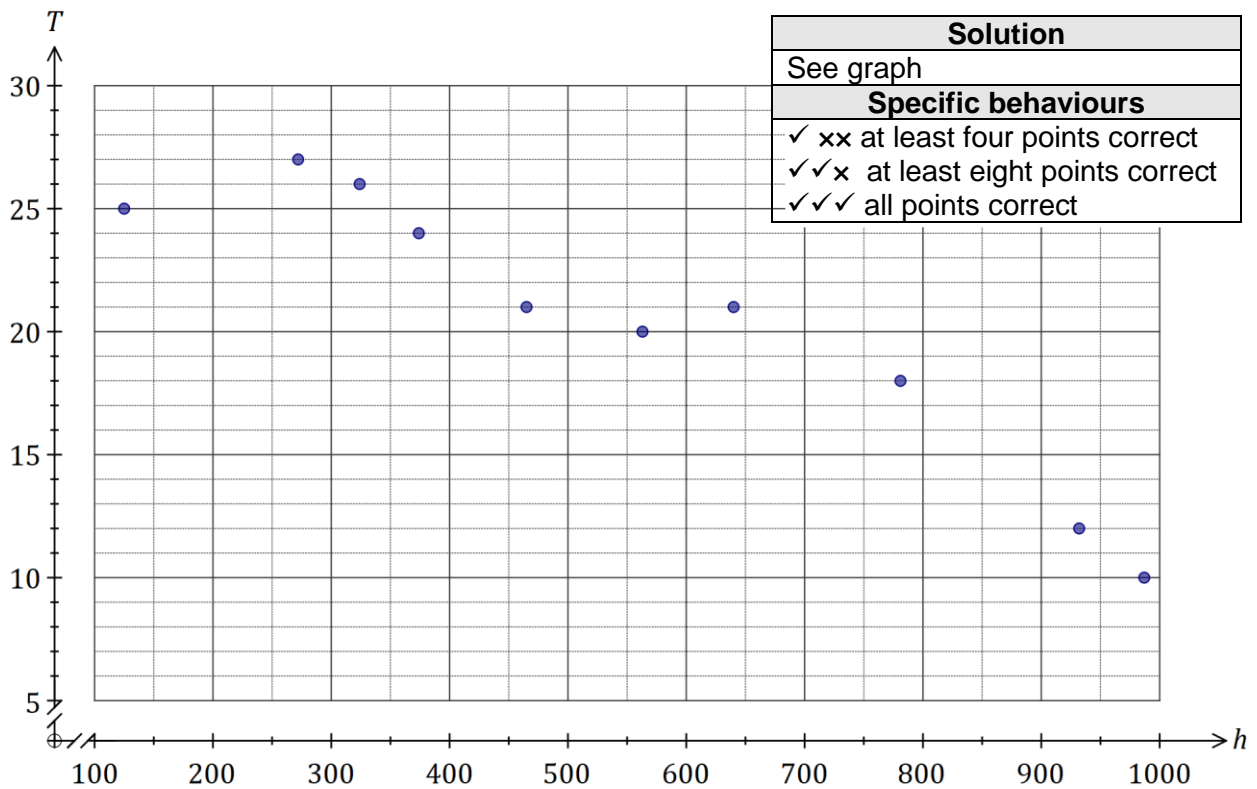| Solution |
|---|
| Roughly twice the percentage of girls prefer to travel by car. Boys are spread evenly across all modes, whereas girls prefer not to use bikes. Bike preference for boys is eight times more than girls. |
| **Specific behaviours** |
| ✓ describes one of the comments above with numbers or %'s |
| ✓ describes two of the comments above with numbers or %'s |

Question 4   **(8 marks)**

The average maximum temperature, T °C, was recorded for ten weather stations, together with the altitude of the station, h metres. The data is shown in the table below.

Altitude, h	987	932	781	640	563	465	374	324	272	125
Temperature, T	10	12	18	21	20	21	24	26	27	25

(a) Construct a scatterplot on the axes below that can be used to identify whether an association exists between altitude and temperature.   (3 marks)

Solution
See graph
Specific behaviours
✓ ✗✗ at least four points correct
✓✓✗ at least eight points correct
✓✓✓ all points correct

(b) Describe the features of the scatterplot that indicate a strong, negative and linear association exists between altitude and temperature.   (2 marks)

Solution
Strong & linear: points lie close to a straight line
Negative: as altitude increases, the temperature decreases
Specific behaviours
✓ strong & linear reason
✓ negative reason

(c) Estimate a value for

(i) the temperature at an altitude of 850 metres.   (1 mark)

Solution
Accept $13.5 \leq T \leq 15.5$
Specific behaviours
✓ within given range

(ii) the value of the correlation coefficient ... variables.   (2 ma...

Solution
Accept $-0.99 \leq r \leq -0.80$
Specific behaviours
✓ negative value
✓ within given range

See next page   SN085-116-3

Question 5 **(5 marks)**

A recursive sequence is defined by the difference equation $aT_{n+1} - bT_n = 12$ and is such that $T_2 = 2$, $T_3 = 4$ and $T_4 = 20$.

(a) Show how to establish the equations $2a - b = 6$ and $5a - b = 3$. (3 marks)

Solution
Using $T_2 = 2$ and $T_3 = 4$
$4a - 2b = 12$
$2a - b = 6$
Using $T_3 = 4$ and $T_4 = 20$
$20a - 4b = 12$
$5a - b = 3$
Specific behaviours
✓ substitutes 2 and 4 correctly
✓ substitutes 4 and 20 correctly
✓ simplifies both to establish the equations

(b) Solve the equations in (a) to determine the values of a and b. (2 marks)

Solution
$5a - b = 3$
$2a - b = 6$
$\quad 3a = -3 \implies a = -1$
$-5 - b = 3 \implies b = -8$
Specific behaviours
✓ solves for a
✓ solves for b

Question 6   **(6 marks)**

A number of people were asked which type of movie genre they preferred to watch from horror, crime and romance. The results, by age of respondent, led to the following table.

	Age of respondent (in years)		
	15 to 34	35 to 54	55 to 74
Horror	64%	40%	26%
Mystery	25%	45%	52%
Romance	11%	15%	22%
Totals	100%	100%	100%

(a)   Name the explanatory variable.   (1 mark)

Solution
Age of respondent
Specific behaviours
✓ Correct variable identified

(b)   Draw a proportional column graph involving three columns of equal height with one column for each of the three categories.   (3 Marks)
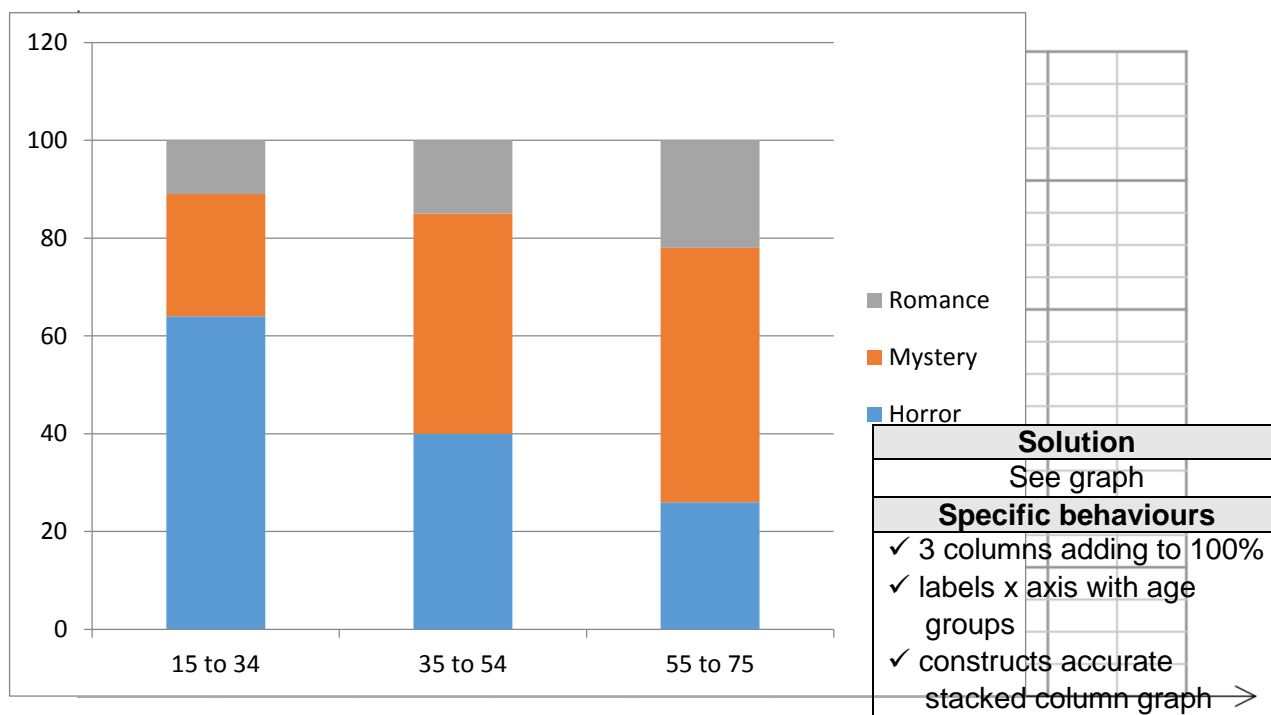
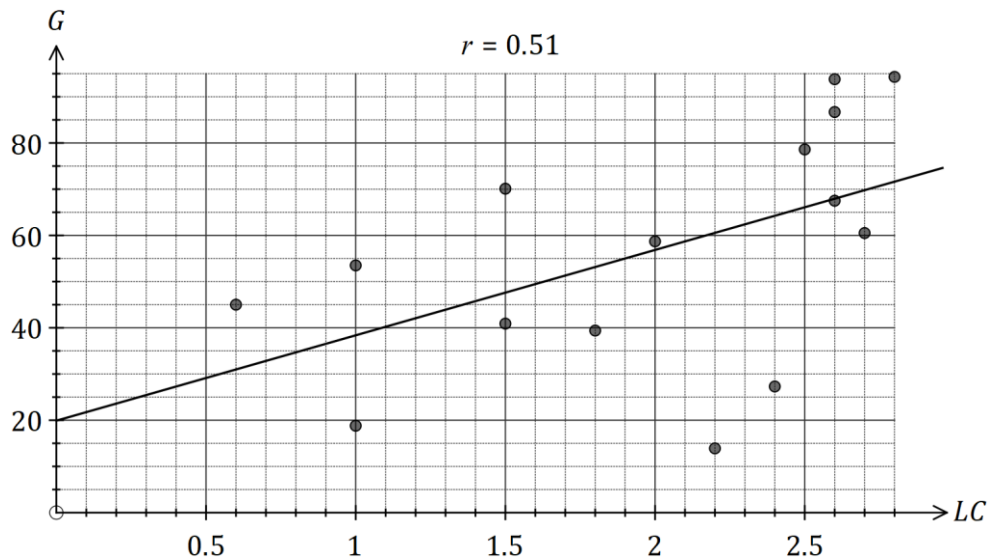| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ 3 columns adding to 100% |
| ✓ labels x axis with age groups |
| ✓ constructs accurate stacked column graph |

(c)   Comment on whether or not there seems to be an association between the variables by describing the association and explaining your reasoning.   (2 marks)

Solution
Yes, there is an association between age and preferred film genre as when the age increases less people watch Horror and more people watch Mystery and Romance.
Specific behaviours
✓ indicates there is an association with a reason
✓ states a reason

See next page

Question 7 **(7 marks)**

A medical study measured the lipoprotein-cholesterol (LC) and ghrelin (G) levels of a group of patients. The results were displayed in the scatterplot below, together with the least-squares line of best fit and the correlation coefficient between the variables.

(a) How many patients in the study with a lipoprotein-cholesterol level of more than 0.75 had a ghrelin level of less than 65? (1 mark)

Solution
8 patients

Specific behaviours
✓ correct number

(b) Determine the upper and lower predicted ghrelin levels for patients with lipoprotein-cholesterol levels between 0.8 and 2.45. (2 marks)

Solution
G between 35 and 65.

Specific behaviours
✓ lower bound, ✓ upper bound

(c) Comment on the claim that a high lipoprotein-cholesterol level causes a patient to have a high ghrelin level. (2 marks)

Solution
The claim is not valid. An observed association does not mean there is a causal relationship between the variables.

Specific behaviours
✓ indicates claim is not valid with a reason
✓ comments on causality

(d) State the number of patients in the study and comment on how the size of the study could influence any explanation for an association between the variables. (2 marks)

Solution
There were 15 patients in the study. This is small number and increases the chance that any association observed may simply be due to coincidence.

Specific behaviours
✓ number in study
✓ comment linking small samples to unreliable outcomes

See next page

Question 8 **(6 marks)**

Given the line of regression for a set of data is $\hat{y} = 10 - 3.4x$, state whether the following are true (T) or false (F).

(a) The predicted value for $x = 5$ is $y = -7$. (1 mark)

Solution
True
Specific behaviours
✓ correct answer

(b) The coefficient of determination is a negative value. (1 mark)

Solution
False
Specific behaviours
✓ correct answer

(c) If the actual y value for when $x = 4$ is 8, then the residual is –3.6. (1 mark)

Solution
False
Specific behaviours
✓ correct answer

(d) If when plotted, the residuals form a random pattern, then this indicates that linear regression is the most appropriate regression to use. (1 mark)

Solution
True
Specific behaviours
✓ correct answer

(e) A prediction for when $x = 19$ would be reliable if the x values for the data plotted ranged from 3 – 17 and the correlation coefficient was –0.87. (1 mark)

Solution
False
Specific behaviours
✓ correct answer

(f) (10, –24) is a point on the line of regression. (1 mark)

Solution
True
Specific behaviours
✓ correct answer

Supplementary page

Question number: _____